$$\frac{[A_{\mathrm{TE}_{mn}}/A_{\mathrm{TM}_{mn}}]_1}{[A_{\mathrm{TE}_{mn}}/A_{\mathrm{TM}_{mn}}]_2} = \frac{[f_{c\mathrm{TM}}/f_{c\mathrm{TE}}]_1{}^2}{[f_{c\mathrm{TM}}/f_{c\mathrm{TE}}]_2{}^2}\,\frac{[f^2 - f^2{}_{c\mathrm{TM}}]_2}{[f^2 - f^2{}_{c\mathrm{TM}}]_1}. \qquad (7)$$

Thus it has been shown that the relative mode amplitudes in the standard waveguide can be readily obtained once the relative amplitudes in the oversize waveguide have been determined.

In addition to the foregoing, we wish to note a correction and an omission. It has been pointed out by M. Sirel of Laboratorie Central Des Ponts Et Chaussees, that reference [11] should have referred to *Electronic and Radio Engineer*, not *Electronics*. Also, due to an oversight, acknowledgment was not made to J. F. Ramsey and Dr. P. A. McInnes, of AIL, for providing the theoretical and measured multimode antenna data used in this effort.

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Constructive Coupling in Directional Couplers

During the course of some diagnostic studies[1] of varactor harmonic generators, we discovered a surprising result using the experimental setup shown in Fig. 1. A signal source is connected through an isolator to port 1 of a 10-dB directional coupler, with the output load connected to port 4 through a slide-screw tuner. Port 2 is internally terminated, and port 3 has connected to it a slide-screw tuner and termination.

By simultaneously adjusting the two slide-screw tuners it was possible to maximize the power delivered to the output load. The reader is invited to pause at this point and estimate the amount of this maximum power for a 10-dB directional coupler. Would you believe a net insertion loss from the generator to the load of 10 dB, 7 dB, 3 dB? The answer, surprisingly, is approximately 3 dB.

This result can be proved using the scattering matrix formalism. For the circuit of Fig. 1 the following matrix equation holds

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & A & 0 & jB \\ A & 0 & jB & 0 \\ 0 & jB & 0 & A \\ jB & 0 & A & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ \Gamma_3 b_3 \\ \Gamma_4 b_4 \end{bmatrix} \qquad (1)$$

where a_k and b_k are the incident and reflected waves on port k, and Γ_3 and Γ_4 are the voltage reflection coefficients of the tuner and load combinations at ports 3 and 4, respectively. B and A are taken to be real by proper

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[1] These studies concerned spurious oscillations resulting when the input circuit presents favorable impedances at two frequencies whose sum is the input frequency.

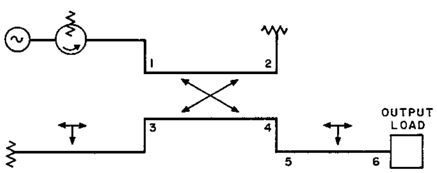

Fig. 1. Experimental setup using a directional coupler.

choice of reference planes. From (1) we have

$$b_3 = A\Gamma_4 b_4 \qquad (2)$$

$$b_4 = jBa_1 + A\Gamma_3 b_3. \qquad (3)$$

Substituting (2) into (3) gives

$$\frac{b_4}{a_1} = \frac{jB}{1 - A^2\Gamma_3\Gamma_4}. \qquad (4)$$

Letting $\Gamma_3 = |\Gamma_3| e^{j\rho_3}$ and $\Gamma_4 = |\Gamma_4| e^{j\rho_4}$ in (4) and noting that $B^2 = 1 - A^2$ gives

$$\frac{|b_4|^2}{|a_1|^2} = \frac{1 - A^2}{1 + A^4|\Gamma_3|^2|\Gamma_4|^2 - 2A^2|\Gamma_3|\,|\Gamma_4|\cos(\rho_3 + \rho_4)}. \qquad (5)$$

Since the tuner is lossless, the power absorbed in the output load is $|b_4|^2(1 - |\Gamma_4|^2)$, and the ratio of load power to generator power is

$$\frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2} = \frac{(1 - A^2)(1 - |\Gamma_4|^2)}{1 + A^4|\Gamma_3|^2|\Gamma_4|^2 - 2A^2|\Gamma_3|\,|\Gamma_4|\cos(\rho_3 + \rho_4)}. \qquad (6)$$

Since $A^2|\Gamma_3|\,|\Gamma_4|$ is positive and less than, or equal to, unity, this is a maximum for

$$\cos(\rho_3 + \rho_4) = 1$$

$$|\Gamma_3| = 1.$$

With these values, (6) becomes

$$\left.\frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2}\right|_{\max} = \frac{(1 - A^2)(1 - |\Gamma_4|^2)}{(1 - A^2|\Gamma_4|)^2}. \qquad (7)$$

When this expression is further maximized with respect to $|\Gamma_4|$, one obtains

$$|\Gamma_4|_{\mathrm{opt}} = A^2 \qquad (8)$$

so that,

$$\left.\frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2}\right|_{\max} = \frac{1}{1 + A^2}. \qquad (9)$$

As the coupling diminishes, $A^2 \to 1$ and the net insertion loss approaches 3 dB. Calculations for 3 dB, 10 dB, and 20 dB directional couplers give net minimum insertion losses of 1.76 dB, 2.79 dB, and 2.99 dB, respectively. Of course, circuit losses will increase these values somewhat, especially for the 20-dB coupler.

The analysis presented above does not lend much physical insight into the mechanism responsible for the unexpected results. A physical explanation is offered as follows. Consider the reciprocal situation where the source and load are interchanged. The two slide-screw tuner probes are inserted deeply so as to form a resonant cavity between them at the operating frequency. For a weakly coupled directional coupler, this resonant cavity has nearly equal waves traveling in both directions. Thus, half the power goes to port 1 and half to port 2, corresponding to a net 3 dB of insertion loss from port 6 to port 1. Application of reciprocity proves the desired result.

Thus we see that the unusual result occurs because of a resonance associated with the in-line arms of a directional coupler. This example shows that one must avoid such resonances in measurement schemes in order to maintain accuracy.

Such a resonance characterizes the behavior of the resonant ring circuit.[2] In the resonant ring circuit, neglecting losses, all of the power ends up in the termination on port 2. In the circuit of Fig. 1, as $A^2 \to 1$ (weak coupling) half of the generator power ends up in the output load connected to port 4, one-fourth of the power is delivered to the port 2 termination, and one-fourth of the power is reflected from port 1.

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[2] H. Golde, "Theory and measurement of Q in resonant ring circuits," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-8, pp. 560–564, September 1960.

Precision Design of Direct Coupled Filters

The direct coupled filter to be discussed in this correspondence consists of a length of transmission line with reflecting obstacles spaced at approximately half wavelength intervals. High-power capabilities can be achieved because the filter can be made without changing the size of the transmission line and, as will be described, without any special tuning devices. The following analysis is applicable for waveguides with inductive obstacles and also to coaxial lines with inductive posts. By duality it is also valid for coaxial or strip line with capacitive gaps.

For obstacles in rectangular waveguide, inductive posts were preferred to irises. The use of a programmed tape controlled milling machine is to be recommended for production runs of this class of filter. Each obstacle used is a pair of posts whose spacing along the guide and from each other can be carefully

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controlled. All the posts in a given filter are of the same diameter.

A paper by Cohn [1] gives formulas for the reactance of each obstacle and their spacing while an article by Smith [2] gives a tabulation of susceptance values calculated from Cohn's formula for almost any value of bandwidth and passband ripple. A recent book [3] treats the fundamental theory of this type of filter.

One equivalent circuit for a symmetrical obstacle in waveguide with properly chosen reference planes is a quarter wavelength of line of a proper impedance level. For a given size of obstacle, a way of finding where the reference planes are located is to put two identical obstacles in the line spaced so that their reflections will cancel each other. The equivalent circuit for the pair would then be a half wavelength long and the displacement of the reference planes from the center line of the obstacle would be half the distance between the obstacles.

One representation of a direct coupled filter at the design frequency is shown in Fig. 1. It represents a direct coupled filter of three stages in waveguide. The equivalent circuit at resonance is given in Cohn's work as a cascade of quarterwave transformers.

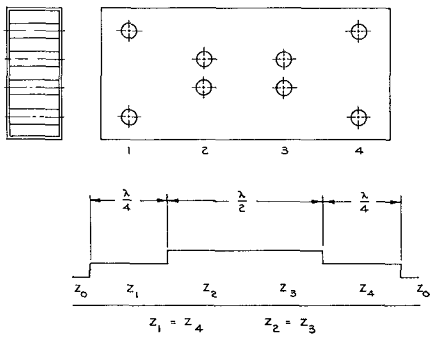

Fig. 1. Three cavity direct-coupled filter and equivalent circuit.

Basically, the method of designing a filter is to select from the works of Cohn or Smith the susceptances of the obstacles to be used in the filter. Then resonant cavities are designed for each size by spacing two of each size of obstacles the proper distance between centerline to give resonance at the design frequency. The filter is then constructed so that *the distance between successive obstacles is the arithmetic mean of the resonant lengths* of the two obstacles at the design frequency. This relation reduces to Cohn's formula when the irises are thin and when their exact magnitude is known. Neither of these conditions is usually met so trimming is required.

The slope of the cavity response near resonance is used as a measure of the susceptance B of the obstacle. The value of Q of a cavity resonator [4] composed of two thin inductive obstacles of susceptance B spaced a distance, s, apart in waveguide is

$$Q_L = \frac{1}{4}\left(\frac{\lambda_g}{\lambda_0}\right)^2\left[-B\sqrt{B^2+4}\tan^{-1}\frac{2}{B} + \frac{2B^2}{\sqrt{B^2+4}}\right]$$

where $\tan 2\pi s/\lambda_g = 2/B$. For inductive discontinuities B has a negative value so $\tan^{-1} 2/B$ is between $\pi/2$ and π. The value of the susceptance jB is assumed to vary as a constant times the guide wavelength. A tabulation of Q as a function of B is given in the Appendix.

To find the spacings of the posts with which to make a filter a series of measurements on a single stage was needed. Rather than solder posts in place in a piece of waveguide, a set of spring loaded sliding posts was used. A special precision section at L-band of 3.25 inches by 6.50 inches ID waveguide 10 inches long was made of $\frac{3}{8}$ inch thick aluminum plates. With the use of a machinist's planar gauge and a depth micrometer, $\frac{3}{8}$ inch diameter posts could be positioned with sufficient accuracy. Good contact was assured with an indium gasket in a groove at each end of the posts.

The four posts were positioned in the guide with a chosen spacing of d, the distance from the side walls to the posts, and s, the distance along the guide between the center lines. The standing wave ratio caused by the resonant stage was measured at a few frequencies on either side of resonance with a well matched load and the Q_L and resonant frequency determined.

A chart was made listing the Q_L and resonant frequency at points whose horizontal coordinate is proportional to the s value and whose vertical coordinate is proportional to the d value as shown in Fig. 2.

After a sufficient number of trials were made, it becomes possible to draw contours of constant resonant frequency and those of constant Q as shown in the sketch. These facilitate the selection of d and s for a desired Q_L and f_0.

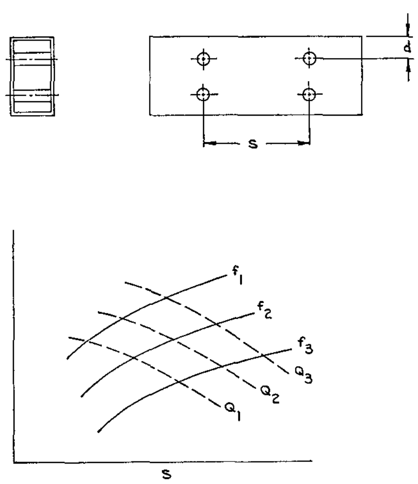

Fig. 2. Single stage resonator and Q-frequency chart.

To determine the value of Q from standing wave measurements the equivalent susceptance of the resonant stage is measured with a matched load. This susceptance is given by $|b| = (r-1)/\sqrt{r}$ where r is the standing wave ratio of the single stage. In this correspondence B represents the equivalent susceptance of a single obstacle and b represents that of a resonant stage. The same formulas for insertion loss and standing wave ratio apply.

A graph or table of the value of b as a function of frequency is made ascribing a positive or negative value of sign to b depending on whether the measurement was made at a frequency greater or less than the resonant frequency. The curve will pass through the X-axis at the resonant frequency. The "half power points" can be found as the values of frequency for which the tangent to the curve at resonance goes to a value of plus and minus 2.

Table I illustrates the method of determining Q [5]. The first column gives the frequency and the second the measured VSWR. The third column gives the equivalent susceptance $|b| = (r-1)/\sqrt{r}$ while the fourth column is the difference between values of the third column. Near resonance the value of b is changed by 0.548 for a variation of 2MHz so the bandwidth (BW) between "half power points" is by simple proportion

$$\frac{0.548}{2.00} = \frac{4.00}{\text{BW}} \qquad \text{BW} = 14.6 \text{ MHz}.$$

Since the resonant frequency is 1243 MHz the Q is 85.

TABLE I

Frequency	VSWR	b	Δb
1238	3.94	−1.48	
			0.597
1240	2.35	−0.883	
			0.579
1242	1.38	−0.324	
			0.548
1244	1.25	+0.224	
			0.536
1246	2.10	+0 760	

Table II was made for a nine stage filter with a passband of 1250 to 1350 MHz to give a high attenuation of 1400 MHz. The complete filter has ten obstacles giving nine resonant circuits. The values of B presented here are those found in Smith for a 20 percent bandwidth filter with 1.2 VSWR in the passband except that the fifth and sixth obstacles were made to be identical with the fourth and seventh for simplicity [6]. The first column gives the order number of the obstacle. The larger susceptances are toward the middle of the filter and the filter is symmetrical end for end. The second column gives the normalized susceptance which is desired for each pair of posts at the design frequency. The third column is the loaded Q (neglecting dissipation losses) which would be achieved for a single resonant cavity using two thin shunt susceptances of this value at a frequency of 1292 MHz the design frequency.

TABLE II

Number	B	Q
5, 6	−5.61	51.27
4, 7	−5.61	51.27
3, 8	−5.20	44.39
2, 9	−3.57	22 06
1, 10	−1.21	3.56

The VSWR of the nine stage filter over the passband of 1250 to 1350 MHz was less than 1.25 and the insertion loss was equal to or less than 0.1 dB. For this filter as a means of comparison the 30 dB points of attenuation were studied. The calculated response from a digital computer program using ideal susceptances of Table II and the experimental results are shown in Table III. Figure 3 shows a plot of the performance. No tuning was used for this filter.

Table IV shows the high-power capacity of models at 5 μs pulse 200 pulses per second with no extra air pressure at room temperature.

TABLE III

	Measured	Calculated
f_1 (30 dB)	1223 MHz	1219
f_2 (30 dB)	1391	1394
f_2-f_1	168	175
$(\frac{1}{2})(f_1+f_2)$	1307	1306
f_3 (Attenuation maximun)	—	89 dB at 1630 MHz
f_4 (30 dB)	1850	1900

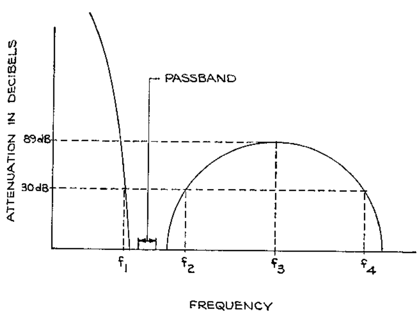

Fig. 3. Attenuation versus frequency for nine resonator filter.

TABLE IV

Frequency	Power (MW)
1250	4.1
1300	7.0
1350	5.5

Appendix

$$\left(\frac{\lambda_0}{\lambda_g}\right)^2 (Q_L) = \frac{1}{4}\left[-B\sqrt{B^2+4}\tan^{-1}\frac{2}{B} + \frac{2B^2}{\sqrt{B^2+4}}\right]$$

$-B$	$(Q_L)\left(\frac{\lambda_0}{\lambda_g}\right)^2$
0	0.000
0.2	0.1778
0.4	0.3998
0.6	0.6694
0.8	0.9892
1.0	1.361
1.2	1.786
1.4	2.265
1.6	2.800
1.8	3.391
2.0	4.039
2.2	4.745
2.4	5.508
2.6	6.331
2.8	7.212
3.0	8.153
3.2	9.154
3.4	10.22
3.6	11.32
3.8	12.52
4.0	13.76
5.0	20.91
6.0	29.60
7 0	39.84
8.0	51.65
9.0	65.03
10 0	79.96
11.0	96.47
12.0	114.5
13.0	134.2
14.0	155.4
15.0	178.2
16.0	202.5
17.0	228.5
18.0	256.0
19 0	285.0
20.0	315.6

Acknowledgment

Among the many people the author wishes to thank for their part in this effort are A. X. Graziano, L. M. Keefer, and P. Kelly.

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Correction to "Comments on Excitation of Spin Waves by Wire Arrays"

In the year since the original comment was written, we have been working on related problems. In October 1966, we discovered an error in the boundary value solution for the case of wires embedded in YIG with dc field perpendicular to the array. In this case the solution is mainly medium-k rather than low-k. We believe our other statements are valid.

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